

AAE635: Applied Microeconomic Theory

Discussion Section 9

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Today's goal

- Review duality in the context of the consumer problem.

Consumer Theory: Primal vs Dual approach

The **primal problem** is utility maximization subject to budget constraint,

$$\begin{aligned} \max_{\mathbf{x}} \quad & u(\mathbf{x}) \\ \text{s.t.} \quad & \mathbf{p} \cdot \mathbf{x} = y \end{aligned} \tag{1}$$

from which we obtain the Marshallian demand $x^*(\mathbf{p}, y)$ and then an indirect utility function,

$$V(\mathbf{p}, y) = \max_x \{U(x) : \mathbf{p}x \leq y, x \geq 0\} = U(x^*(\mathbf{p}, y)) \tag{2}$$

If we start from an indirect utility function, we can obtain Marshallian demand functions through Roy's identify:

$$x_i^*(\mathbf{p}, y) = -\frac{\frac{\partial V}{\partial p_i}}{\frac{\partial V}{\partial y}} \tag{3}$$

The **dual problem** is expenditure minimization subject to utility level,

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{p} \cdot \mathbf{x} \\ \text{s.t.} \quad & u(\mathbf{x}) = U \end{aligned} \tag{4}$$

from which we obtain the Hicksian demand $x^c(\mathbf{p}, U)$ and then an indirect expenditure function,

$$E(p, \bar{U}) = \min_x \{ \mathbf{p}\mathbf{x} : U(\mathbf{x}) \geq \bar{U}, \mathbf{x} \geq 0 \} = \mathbf{p}\mathbf{x}^c(\mathbf{p}, \bar{U}) \quad (5)$$

If we start from an indirect expenditure function, we can obtain Hicksian demand functions through Shephard's lemma:

$$x_1^C(\mathbf{p}, \bar{U}) = \frac{\partial E}{\partial p_i} \quad (6)$$

Slutsky Equation. When price of a good increases, $p_1 \rightarrow p'_1$, there are two effects.

- Substitution effect: holding the utility constant, verify effect of price ratio.
- Income effect: holding price ratio, verify effect of budget level.

Let's first see this in algebra. Note the identity from the primal-dual problems,

$$x_i^c(\mathbf{p}, U) \equiv x_i^*(\mathbf{p}, E(\mathbf{p}, U)) \quad (7)$$

from which we can derive the Slutsky equation

$$\frac{\partial x_i^*(\mathbf{p}, y)}{\partial p_j} = \frac{\partial x_i^c(\mathbf{p}, U)}{\partial p_j} - \frac{\partial x_i^*(\mathbf{p}, y)}{\partial y} x_j^*(\mathbf{p}, y), i, j = 1, 2, \dots, n \quad (8)$$

Slutsky Equation.

$$\underbrace{\frac{\partial x_i^*(\mathbf{p}, y)}{\partial p_j}}_{\text{Marshallian price effect}} = \underbrace{\frac{\partial x_i^c(\mathbf{p}, U)}{\partial p_j}}_{\text{Hicksian price effect}} - \underbrace{\frac{\partial x_i^*(\mathbf{p}, y)}{\partial y} x_j^*(\mathbf{p}, y)}_{\text{Income effect}}$$

- **Marshallian price effect:** "When a price increases, how do Marshallian demands respond".
- **Hicksian price effect (or, substitution effect):** Intuitively, substitution effects capture people's incentive to buy more of goods with lower prices and less of goods with higher prices, ignoring income effects below.
- **Income effect:** Notice that when a price increases, the buying power of income is in effect reduced, even though actual income does not change. Income effects then capture how this change in buying power affects demands.

Practice Question

1. Consider a household choosing two consumption goods $\mathbf{x} = (x_1, x_2)'$. The household has preferences represented by the CES utility function:

$$U(x_1, x_2) = \left(x_1^{\frac{1}{2}} + x_2^{\frac{1}{2}}\right)^2$$

The household budget constraint is $p_1x_1 + p_2x_2 \leq y$, where $y > 0$ is his income and $\mathbf{p} = (p_1, p_2)'$ is the market price vector for $\mathbf{x}$

- (a) Set up the household's utility maximization problem. Does CQ holds? Why?
- (b) Use the Lagrangean method to find the utility maximization decision rules for household consumption.
- (c) Derive the Indirect Utility Function $V(\mathbf{p}, y)$ based on the result from (b). (Hint: derive the Marshallian demand functions)
- (d) Verify that Roy's identity holds.
- (e) Set up the household's expenditure minimization problem. Does CQ holds? Why?
- (f) Use the Lagrangean method to find the expenditure minimization decision rules for household consumption
- (g) Derive the Expenditure Function based on (f).
- (h) Verify that Shephard's lemma holds.
- (i) Find the household's Indirect Utility Function $V(\mathbf{p}, y)$ starting from the expression for $E(\mathbf{p}, U)$
- (j) Find the household's Expenditure Function $E(\mathbf{p}, U)$ starting from the expression for $V(\mathbf{p}, y)$.