

AAE635: Applied Microeconomic Theory

Discussion Section 6

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Today's goal

- Understand how to solve constrained optimization problems using the Lagrangian method and interpret the economic meaning of the multiplier.
- Learn and interpret key duality results: Envelope Theorem, Hotelling's Lemma, and Shephard's Lemma.

Practice Question

A farmer has 72 acres of arable land (A). The production functions for organic and conventional vegetable production are given by

$$y_o = A_o^{0.6} \quad \text{and} \quad y_c = A_c^{0.8},$$

respectively. The net profit from organic production is \$400 per unit of output, while the net profit from conventional production is \$300 per unit of output. The farmer wishes to maximize total profits and must decide how many acres to allocate to organic production (A_o) and to conventional production (A_c).

1. Set up the farmers constrained optimization problem following the Lagrangian approach. Does the constraint qualification (CQ) hold?
2. Solve the first-order necessary conditions (FONC) to determine the farmers optimal land allocation.

3. Notice that the farmer chooses to allocate fewer acres to organic production than to conventional production, despite the fact that organic farming is more profitable per unit. Use the concept of marginal productivity to argue this.

Duality

We start from an indirect objective function, $V(\alpha)$.

$\Rightarrow$ Analyze properties of $x^*(\alpha)$ and $V(\alpha)$

1. **Envelope Theorem:** The derivative of an indirect objective function with respect to parameters equals the partial derivative of a Lagrangian function:

$$\frac{dV(\alpha)}{d\alpha} = \frac{\partial L}{\partial \alpha}$$

This is used to obtain the three results below.

2. **Interpretation of (λ) , the Lagrange multiplier:**

Suppose that

- the objective function f does not depend on a parameter α , and
- the constraint function is $h(x, \alpha) = \alpha - g(x)$

Then, the Lagrangian multiplier is the derivative of an indirect objective function with respect to α : $\frac{dV(\alpha)}{d\alpha} = \lambda^*$.

How is it important? Remember the cost minimization problem in which the objective function is $f(x) = w \cdot x$ and the constraint function is $h(x, \bar{y}) = \bar{y} - g(x)$.

Then, the interpretation of the Lagrangian multiplier is that “the increase in total cost in response to a marginal increase in the target production”.

3. **Hotelling’s Lemma:** A partial derivative of an indirect profit function with respect to the price of netput j is a *netput function* of j :

$$\frac{\partial \pi(p)}{\partial p_j} = y_j(p).$$

So, if we take a partial derivative of an indirect profit function with respect to an *output price*, then we get an **output function**.

If we take a partial derivative of an indirect profit function with respect to an *input price*, then we get the **negative of an input function**.

4. **Shephard's Lemma:** A partial derivative of an indirect cost function with respect to the price of input j is an *input demand function* of j :

$$\frac{\partial C(w, y)}{\partial w_j} = x_j^c(w, y).$$

So, if we have an indirect cost function, we can take a derivative with respect to an input price and obtain an input function.