

AAE635: Applied Microeconomic Theory

Discussion Section 2

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Today's goal

- Review the profit maximization problem of a competitive firm using first-order and second-order conditions.
- Distinguish between necessary and sufficient conditions for an interior optimum.

Practice Questions

1. A competitive firm has the following revenue function:

$$R(x) = 300x - 5x^2,$$

where x is the amount of input used, and the price of input is $w > 0$.

- (a) Write down the firm's profit function $\pi(x) = R(x) - wx$.
- (b) Derive the first-order condition (FONC) for the profit-maximizing level of input, $x^*(w)$.
- (c) Verify whether the second-order sufficient condition (SOSC) is satisfied.
- (d) Interpret how the optimal input demand $x^*(w)$ changes when w increases.

2. Suppose a firm's profit function is given by:

$$\pi(x) = x^3 - 6x^2 + 9x, \quad x \geq 0.$$

- (a) Compute $\pi'(x)$ and find the critical points.
- (b) Apply the first-order necessary condition (FONC) to identify candidate optima.
- (c) Use the second derivative to check the second-order conditions (SONC and SOSC).
- (d) Determine whether each critical point corresponds to a maximum, a minimum, or neither.
- (e) Briefly explain why FONC alone is not enough to establish whether a point is an optimum.