

AAE635: Applied Microeconomic Theory

Discussion Section 13

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Today's goal

- To review the second half of the course.

Consumer Theory

1. If function u is **quasi-concave**, then FONCs are sufficient to identify a global interior solution x^* to the utility maximization problem.
2. u is quasi-concave **iff** the preference set is convex.
 - Let x_1, x_2 be two consumption bundles. A preference relation $\succeq$ is called convex if $\forall x_1, x_2 \in X$ where $x_2 \succeq x_1$, then for every $\theta \in [0, 1]$, $\theta x_1 + (1 - \theta)x_2 \succeq x_1$. If a bundle x_2 is preferred over bundle x_1 , then any mix of x_2 with x_1 is still preferred over x_1 .
3. u is quasi-concave **iff** $\forall x_1, x_2 \in X, u(\theta x_1 + (1 - \theta)x_2) \geq \min\{u(x_1), u(x_2)\}$.
4. Quasi-concavity on the utility function implies diminishing marginal rate of substitution. That is, moving along the indifference curve, MRS decreases (the slope becomes flatter).
5. **Non-satiation assumption:** The more is the better. Household can always increase its utility by increasing consumption of some commodity.
 - An implication is that the budget constraint is necessarily binding at the optimum x^* .

- The Lagrangian multiplier λ is always positive. It measures the change in utility corresponding to a change in income.

6. Primal and dual consumer problems

- Utility maximization (primal): $\max_{x \geq 0} u(x) \quad \text{s.t.} \quad p \cdot x \leq y$
- Expenditure minimization (dual): $\min_{x \geq 0} p \cdot x \quad \text{s.t.} \quad u(x) \geq \bar{u}$
- Under standard assumptions (monotonicity, convexity), both problems generate the same optimal consumption choices.

7. Value functions and duality

- Indirect utility function: $v(p, y) = \max_x \{u(x) : p \cdot x \leq y\}$
- Expenditure function: $e(p, \bar{u}) = \min_x \{p \cdot x : u(x) \geq \bar{u}\}$
- Duality relations: $v(p, e(p, u)) = \bar{u}, \quad e(p, v(p, y)) = y$

8. Demand functions

- Marshallian demand $x(p, y)$ solves the primal problem.
- Hicksian demand $x^c(p, u)$ solves the dual problem.
- Both represent the same preferences in different spaces: (p, y) vs. (p, u) .

9. Envelope results

- Roy's Identity:

$$x_i(p, y) = - \frac{\partial v(p, y) / \partial p_i}{\partial v(p, y) / \partial y}$$

- Shephard's Lemma:

$$x_i^c(p, u) = \frac{\partial e(p, u)}{\partial p_i}$$

10. Slutsky Equation:

$$\frac{\partial x^M}{\partial p_i}(p_i, p_j, y) = \frac{\partial x^c}{\partial p_i}(p_i, p_j, \bar{U}) - \frac{\partial x^M}{\partial y}(p_i, p_j, y) x^c(p_i, p_j, \bar{U})$$

11. Cournot aggregation restriction:

$$x_1^* + p_1 \frac{\partial x_1^*}{\partial p_1} + p_2 \frac{\partial x_2^*}{\partial p_1} = 0$$

12. **Engel aggregation restriction:**

$$p_1 \frac{\partial x_1^*}{\partial y} + p_2 \frac{\partial x_2^*}{\partial y} = 1$$

13. **Compensating Variation (CV)** is the compensation for the price change.

$$CV = E(p_1, u_0) - y = E(p_1, u_0) - E(p_0, u_0) = E(p_1, u_0) - E(p_1, u_1)$$

- CV uses the original utility as benchmark.
- How much is needed to keep the original utility.

14. **Equivalent Variation (EV)** is the income change equivalent to the price change.

$$EV = E(p_1, u_1) - E(p_0, u_1) = E(p_0, u_1) - E(p_0, u_0)$$

- CV uses the new utility as benchmark.
- If price doesn't change, what's the equivalent income change that gives the same new utility.

15. **Benefit function:** It measures the household willingness-to-pay to obtain X starting from utility $\bar{U}$.

$$B(X, \bar{U}) \equiv \max_{\beta} \{ \beta : U(X - \beta g) \geq \bar{U}, (X - \beta g) \geq 0 \}$$

16. An allocation is **Pareto efficient** iff it is feasible and there is no other feasible allocation that can make one person better off without making any other worse off.

- (a) **Contract curve:** all Pareto efficient allocations.
- (b) **Core:** all allocations that are Pareto efficient and individually rational.