

AAE635: Applied Microeconomic Theory

Discussion Section 11

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Today's goal

- Review the benefit function.
- Solve the household production and time allocation problem.

Benefit Function

The benefit function measures the household willingness-to-pay to obtain X starting from utility $\bar{U}$.

$$B(X, \bar{U}) \equiv \max_{\beta} \{ \beta : U(X - \beta g) \geq \bar{U}, (X - \beta g) \geq 0 \}$$

This suggests measuring the welfare effects of a change from X_a to X_b by the function

$$W(X_a, X_b, \bar{U}) = B(X_b, \bar{U}) - B(X_a, \bar{U})$$

The household is said to be better off (worse off) by the change from X_a to X_b if $W(X_a, X_b, \bar{U}) > 0$ (< 0).

Practice Questions

1. **Benefit Function.** Consider a consumer with the utility function $u(x_1, x_2) = x_1 x_2$. Recall that the benefit function is defined as

$$B(x_1, x_2, \bar{u}) = \max_{\beta} \{ \beta \in \mathbb{R} \mid u(x - \beta g) \geq \bar{u}, (x - \beta g) \geq 0 \}.$$

Suppose that the reference consumption bundle is $(\bar{x}_1, \bar{x}_2) = (10, 3)$ so that $\bar{u} = 10 \times 3 = 30$. If the reference commodity bundle is $g = (1, 0)$, find $B(x_1, x_2, \bar{u}) = B(15, 6, 30)$ and $B(x_1, x_2, \bar{u}) = B(4, 10, 30)$.

2. **Household Production Problem.** Your household is running a family business. And in total you have 24 hours. You can devote your time into 3 categories:

- $t_h > 0$: work at home for your family business
- $t_w > 0$: work outside home at wage rate $w = 5$
- $\ell > 0$: leisure time

From the time you devote yourself in working at home, you can produce marketable good $x > 0$, with price $p_x = 5$. Your total income comes from your family business and working outside your home. You use the income to consume $c > 0$ units of goods at price $p_c = 2$. Normalize your capital into 1, then your family production function h is: $x = 4t^{0.5}$. Your utility function is $U(c, \ell) = \log c + \log \ell$.

- (a) Write down the optimization problem.
- (b) Solve for the optimal level of time working at home.
- (c) Solve for the optimal level of time of leisure and your optimal utility.
- (d) Assume now the government is now encouraging the family business by providing a subsidy to the family-produced marketable good, which is equivalent to increase p_x to $(1 + \tau)p_x$, where $\tau > 0$. In order to keep our solution remaining in the interior of our feasibility set, τ needs to be less than _____. Discuss how this will change your optimal time allocations.