

AAE635: Applied Microeconomic Theory

Discussion Section 10

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Today's goal

- Review duality in the context of the consumer problem.

The Slutsky Equation

Assume there are two goods: i and j . We are interested in knowing how the consumption of good i changes when its price changes. We know that

$$x^c(p_i, p_j, \bar{U}) = x^M(p_i, p_j, E(p_i, p_j, \bar{U}))$$

Let's derive that with respect to p_i .

$$\Rightarrow \frac{\partial x^c}{\partial p_i}(p_i, p_j, \bar{U}) = \frac{\partial x^M}{\partial p_i}(p_i, p_j, y) + \frac{\partial x^M}{\partial y}(p_i, p_j, y) \frac{\partial E}{\partial p_i}(p_i, p_j, \bar{U})$$

We know that

$$\frac{\partial E}{\partial p_i}(p_i, p_j, \bar{U}) = x^c(p_i, p_j, \bar{U}) \quad (\text{Shephard's Lemma})$$

$$\Rightarrow \frac{\partial x^c}{\partial p_i}(p_i, p_j, \bar{U}) = \frac{\partial x^M}{\partial p_i}(p_i, p_j, y) + \frac{\partial x^M}{\partial y}(p_i, p_j, y) x^c(p_i, p_j, \bar{U})$$

$$\Rightarrow \frac{\partial x^M}{\partial p_i}(p_i, p_j, y) = \frac{\partial x^c}{\partial p_i}(p_i, p_j, \bar{U}) - \frac{\partial x^M}{\partial y}(p_i, p_j, y) x^c(p_i, p_j, \bar{U})$$

This is the Slutsky equation.

$\frac{\partial x^c}{\partial p_i}(p_i, p_j, \bar{U})$ is the **substitution effect**

$-\frac{\partial x^M}{\partial y}(p_i, p_j, y) x^c(p_i, p_j, \bar{U})$ is the **income effect**

Welfare Analysis

To measure consumer's welfare change due to price change, we consider three measurements.

1. **Consumer Surplus (CS)** is the monetary gain/loss obtained by consumers. $\Delta CS = \int_{p_0}^{p_1} x^*(p, y) dp$

- CS takes into account the income effect.
- CS involves utility change (no proper benchmark).

2. **Compensating Variation (CV)** is the compensation for the price change.

$$CV = E(p_1, u_0) - y = E(p_1, u_0) - E(p_0, u_0) = \int_{p_0}^{p_1} x^c(p, u_0) dp$$

- CV uses the original utility as benchmark.
- How much is needed to keep the original utility.

3. **Equivalent Variation (EV)** is the income change equivalent to the price change.

$$EV = E(p_1, u_1) - E(p_0, u_1) = \int_{p_0}^{p_1} x^c(p, u_1) dp$$

- CV uses the new utility as benchmark.
- If price doesn't change, what's the equivalent income change that gives the same new utility.

Consider the case of normal goods.

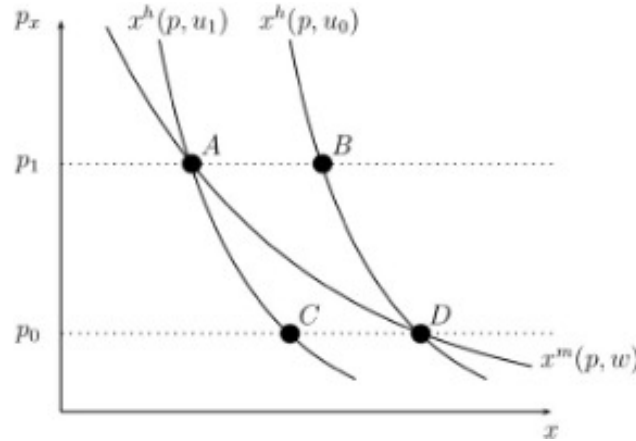
- If price goes up $p_1 > p_0$, then

$$|EV| < |\Delta CS| < |CV|$$

- If price goes down $p_1 < p_0$, then

$$|CV| < |\Delta CS| < |EV|$$

Figure 1: Changes in prices



Practice Question

Suppose that you are gaining utility from ordinary tea x_1 and specialty coffee x_2 with $u(x_1, x_2) = x_1 x_2^2$. In order to get the specialty coffee that you really need, you have a long-term contract with a coffee company. The tea price is $p_1 = 1$, and the coffee price is $p_2 = 5$, and your budget for tea and coffee is 90. Using the expenditure function $E(p_1, p_2, u)$ and the indirect utility function $V(p_1, p_2, I)$, answer the following questions. Assume $V(p_1, p_2, I) = \frac{4I^3}{27 p_1 p_2^2}$ and $E(p_1, p_2, u) = \left(\frac{27}{4}\right)^{1/3} u^{1/3} p_1^{1/3} p_2^{2/3}$.

1. An unexpected technology innovation suddenly occurs. If the technology is adopted, the price of coffee will get lower at 3. Now, you want the company to adopt it.
 - (a) How much are you willing to pay the firm in order to make the company use it? How does this relate to CV or EV?
 - (b) According to the contract between you and the firm, which is very favorable to you, the firm must use the most advanced technology and must bear all of the fixed cost to switch technology. The firm does not want to adopt the newly invented technology because the switching cost is very high. In this case, how much does the company need to pay you in order to obtain your agreement that the firm continue to use the old technology? How does this relate to CV or EV?

2. Due to an unexpected increase in the international coffee price, the company wants to increase the price from 5 to 10.
- (a) According to the contract, the price cannot be changed unless both the parties agree. In order to obtain your agreement on increasing the price to 10, how much does the company need to pay you at minimum? How does this relate to CV or EV?
 - (b) The contract provides that the company has perfect freedom to change the price. In this case how much at maximum are you willing to pay to the company in exchange of keeping the same price of 5? How does this relate to CV or EV?